and axial reflection symmetry about the plane $z=z_0$)

$$F(r, 2\theta_0 - \theta, 2z_0 - z) = F(r, \theta, z).$$

5) Combined angular rotation symmetry (about the structural axis) and axial reflection symmetry (about the plane $z=z_0$)

$$F(r, \theta + \psi, 2z_0 - z) = F(r, \theta, z).$$

6) Screw symmetry (rotation about the structural axis combined with translation along it)

$$F(r, \theta + \psi, z + \delta) = F(r, \theta, z).$$

7) Glide symmetry (angular reflection about the half plane $\theta=\theta_0$ combined with translation of one-half period along the structural axis)

$$F(r, 2\theta_0 - \theta, z + p/2) = F(r, \theta, z).$$

Some important consequences of these symmetries are discussed in the article by Crepeau and McIsaac [9].

Michael J. Gans
Dept. of Electl. Engrg.
University of California
Berkeley, Calif.

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Field Measurements in a Small Cross Section Guide Loaded with Magnetized Ferrite

By solving the Maxwell's equations for a rectangular guide partially filled with magnetized ferrite one finds that in particular conditions to be specified later for the geometry and the applied magnetic field only modes with phase velocity directed in one sense can propagate. This is the basis for the so-called thermodynamical paradox. The present work was carried out in order to experimentally investigate the microwave e.m. field in such a structure and compare the results with the theoretical predictions which we summarize here briefly.

Manuscript received November 6, 1964; revised December 29, 1964. The work reported here was partly supported by the Consiglio Nazionale delle Ricerche, Italy.

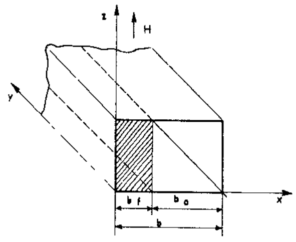

Fig. 1. Reference system.

We assume the configuration and notations shown in Fig. 1 (indefinite in the y direction). We assume for the microwave field a dependence $\exp(-k_x x - +k_y y - k_z z)$ with the coordinates.

The characteristic equation of the system was numerically solved by Barzilai and Gerosa for a number of cases ([1], [2], [3]) and the following main results were established:

1) For a given ferrite the solutions of the characteristic equation depend on the ratio ω/ω_0 between the working frequency and the resonant frequency $\omega_0=\gamma H$.

2) For $0<\mu_1<{}_{m2}$ (i.e., for

$$\sqrt{1-\frac{\omega_m}{\omega_0}} < \frac{\omega}{\omega_0} < 1 + \frac{\omega_m}{\omega_0},$$

assuming the tensor permeability in the form:

$$\mu = \mu_0 \begin{vmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

with

$$\mu_1 = 1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2} \quad \text{and} \quad \mu_2 = \frac{\omega\omega_m}{\omega_0^2 - \omega_m^2}$$

where $\omega_m = 4\pi\gamma\mu_0$), a class of unidirectional propagating modes is found (the ferrite-dielectric and ferrite-metal modes of Seidel and Fletcher [6]). It is possible to choose the dimensions of the guide and the thickness of the ferrite slab in such a way that, within a given range of the applied magnetic field, these unidirectional modes be the only propagating modes, all higher order modes (the ferrite-guided modes following Lax's classification [5][1]) being under cutoff. The situation previously described gives rise to the so-called thermodynamical paradox, because all the propagating modes (nonattenuated for lossless ferrite) have phase velocity in one direction and no mode propagates in the opposite direction.

We recall that this result applies to an indefinite structure and does not include all the modes with complex propagation constants which exist also for lossless ferrite, and are actually the only existing modes for lossy ferrite.

In order to verify these theoretical results we considered a small cross section guide with dimensions 10.25 by 5.45 mm loaded with a ferrite slab 1.5 mm thick against the side wall. The ferrite was transversely magnetized by an applied dc magnetic field ranging in value up to 4400 Oe. The microwave field was investigated by plunging a small dipole in the guide and displacing it in all coordinate directions, making use of some different mechanical arrangements (described in Bujatti [7]). Different ferrite slabs and different lengths of the slab were used but no appreciable modification was found in the field pattern. Also, more accurate polishing of the ferrite slab did not affect the measurements, i.e., the effects of accidental imperfections on the surface of the slab were negligible. Finally, the field pattern is not affected by the termination, for a ferrite slab sufficiently long, for any value of the applied magnetic field. Once this was established the guide was left most of time without any termination and the probe introduced from the open end. The measurements were repeated for different values of the applied magnetic field with a fixed working frequency equal to 9700 MHz and the following results were established:

1) No output can be detected at the end of the structure except when the applied magnetic field H assumes a value $H'<H<H''$ (with $H'=1500$ Oe and $H''=2800$ Oe for R4 Ferramic and a working frequency equal to 9700 MHz as previously stated) if the ferrite slab is sufficiently long (more than 3–4 cm).

2) For applied magnetic field values $H'<H<H''$ the microwave field is exponentially decreasing in the x direction at a rate varying with the applied magnetic field, reaching a maximum of about 8 dB/mm in the center of the range $H'-H''$, and a value of about 3.5 dB/mm at the boundaries of the range $H'-H''$. The microwave field is a surface wave guided by the ferrite [Fig. 2(b)]. By rotating the probe around the x axis, from the z to the y direction the presence of a y component of the microwave field was found, which should not be there if only the zero-order mode was excited.

For applied field values $H'<H<H''$, the dependence on y always shows oscillations of the microwave field along the length of the guide (Figs. 4, 5, and 6) which disappear for applied magnetic fields out of the range $H'-H''$ (Fig. 3. Figures 4 and 5 show how the pattern changes by moving the probe along the ferrite slab (z direction) and away from the ferrite slab (x direction).

Finally, always for applied field values $H'<H<H''$, the dependence on z is shown in Fig. 6 and again oscillations are detected showing the presence of higher order modes. A tentative modal analysis mainly suggests the presence of the second- and four-order harmonics.

3) For applied magnetic fields $H<H'$ (including all negative values) and $H>H''$ (up to the highest value tested equal to 4400 Oe) the dependence of the microwave field on x, normally to the ferrite slab, was found to be sinusoidal; the dependence on y, along the length of the guide, was found exponentially decreasing and the field was found to be constant with z. The overall configuration is the same expected for a TE_{10} in a guide under cutoff loaded with a slab of dielectric material having relative dielectric constant equal to 11 and scalar permeability varying around one depending on the applied magnetic field. The attenuation in the y direction depends on the value of the applied magnetic field as expected by the

[1] See Lax and Button [5], p 395.

variation of the permeability and is the same for the same absolute value of H, no matter what the direction (provided of course $|H| < H'$ or $|H| > H''$).

We now compare the experimental results with the theoretical predictions. We observe first that from experimental values reported in Bujatti [4] for R4 Ferramic, we have $\mu_1 = \mu_2$ at $H/f = 148$ Oe/KMHz. This value agrees closely enough with the value $H'/f = 155$ Oe/KMHz. From the theoretical expressions of the permeability components, taking $M_0 = 1780$ gauss and $g = 2.05$ (which are typical values for R4 Ferramic[2] we obtain $\mu_1 = \mu_2$ for $H/f = 168$ Oe/KMHz. Less satisfactory is the agreement between $H'' = 2800$ Oe and the value of H/f corresponding theoretically to $\mu_1 = 0$, which is $H/f = 240$ Oe/KMHz; we do not know of any sufficiently accurate experimental value on this point. We then consider a numerical case theoretically solved and reported in Brazilai and Gerosa [2][3] which is quite similar to our experimental situation. The geometry is the same and the only significant difference is in the value of the saturation magnetization M_0 (assumed equal to 3000 gauss in Brazilai and Gerosa [2]) and, therefore, in the values of $\tau (\tau = \omega/\gamma H)$ and $\rho (\rho = \mu_0/\mu_0 H)$. Taking a rough account of the different value of M_0 the theoretical case reported in Brazilai and Gerosa [2] approximately corresponds to our experimental situation for $H = 2000$ Oe (the correspondence is based on the same values of τ and ρ, but obviously both values cannot be matched exactly by adjusting the only value of H, and $H = 2000$ Oe represents a optimum compromise). The theoretical analysis gives for the attenuation in the x direction of the zero-order mode a value $k_x/\omega\sqrt{\mu_0\epsilon_0} = 1.7$ and for the propagation constants of the zero- and second-order modes $k_{y0}/\omega\sqrt{\mu_0\epsilon_0} = j1.9$ and $k_{y2}/\omega\sqrt{\mu_0\epsilon_0} = j4.4$. We find experimentally [see Fig. 2(b)] and attenuation along x corresponding to $k_x = 4.2$ cm^{-1}, i.e., to $k_x/\omega\sqrt{\mu_0\epsilon_0} = 2.0$. This value has to be compared with the theoretical attenuation for the zero-order mode because the higher the order of the mode the higher the attenuation in the x direction can be shown to be and, therefore, only the zero order is significant here. Given the approximation in the correspondence between the theoretical and the experimental situation, the agreement can be considered satisfactory.

As far as the dependence on y and z are concerned we suggest that the oscillations detected are due to the superposition of higher order modes on the fundamental one. This hypothesis is supported by the following arguments:

1) The oscillations were proved not to be due either to the roughness of the surface or to reflections at the termination.

2) The value of the propagation constant along y for a given ferrite-dielectric or ferrite-metal mode decreases with increasing magnetic field (see, Seidel and Fletcher e.g., [6]); therefore, since taking $k_y = j\beta$, and with the choice of the functional dependence on the coordinates previously stated. k_x^2

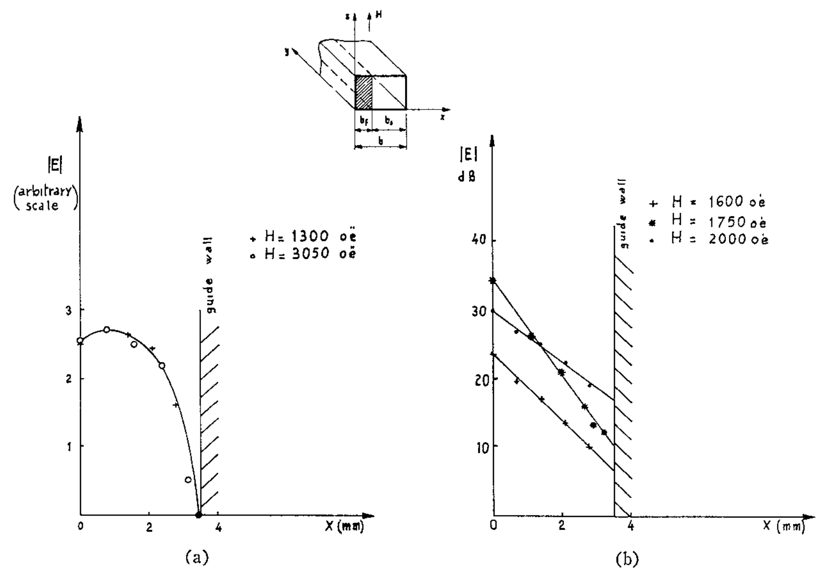

Fig. 2. Microwave electric field along z as a function of x for different values of H.

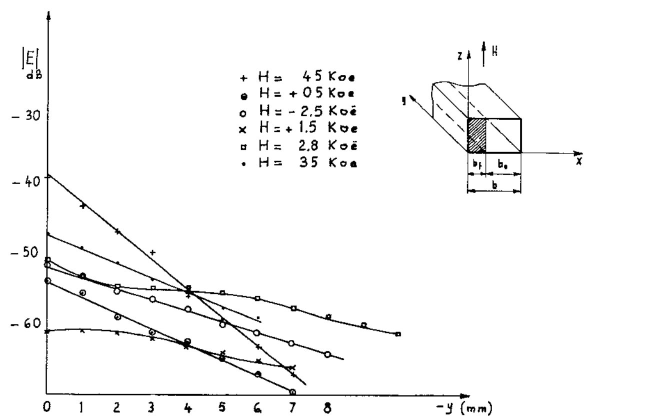

Fig. 3. Microwave electric field along z as a function of y for different values of H within the ranges $H < 1500$ Oe and $H > 2800$ Oe.

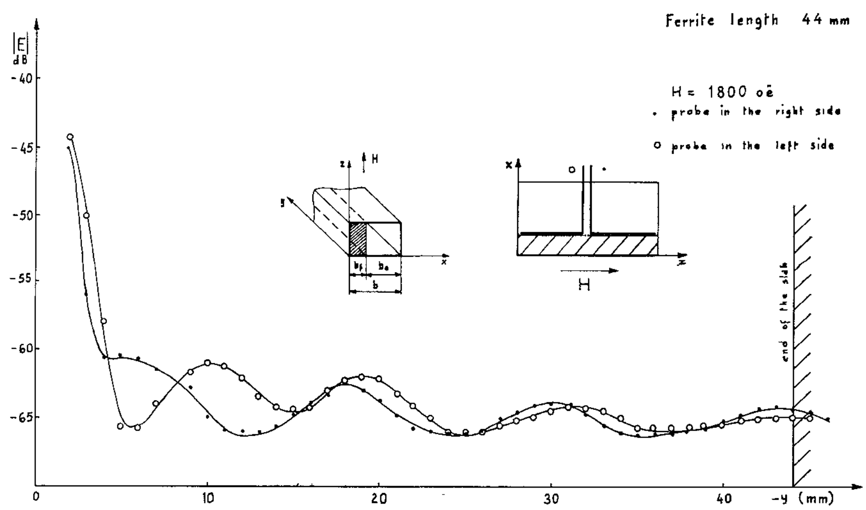

Fig. 4. Microwave electric field along z as a function of y for two different values of z and for $H = 1800$ Oe.

[2] See Lax and Button [5], p 705.
[3] See Fig. 6 of Brazilai and Gerosa [2].

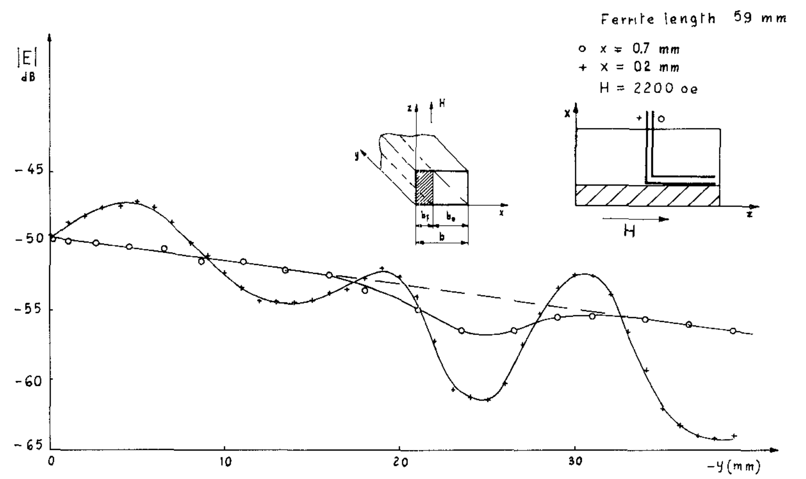

Fig. 5. Microwave electric field along z as a function of y for two different values of x and for $H = 2200$ Oe.

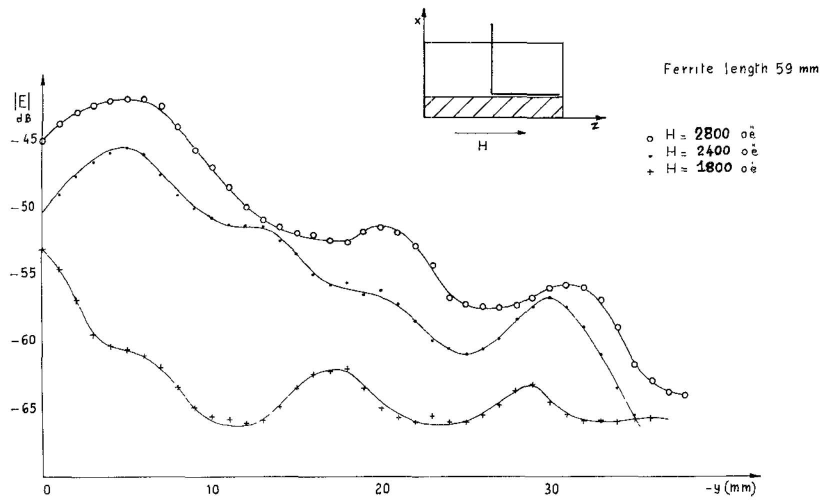

Fig. 6. Microwave electric field along z as a function of y for different values of H within the range 1500 Oe $< H >$ 2800 Oe.

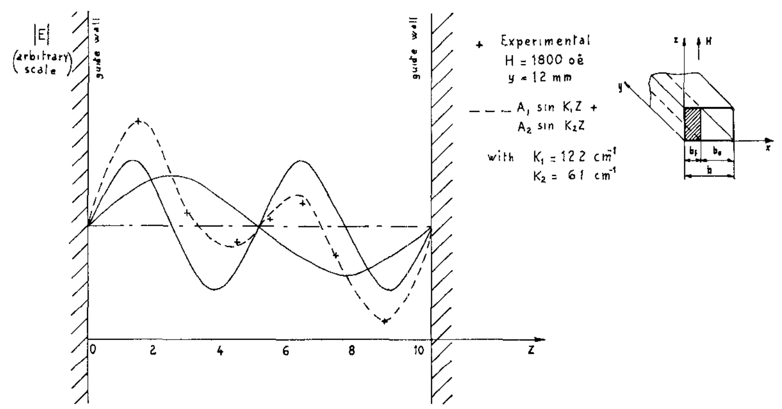

Fig. 7. Microwave electric field z as a function of z for $H = 1800$ Oe.

$= \omega^2 \epsilon_0 \mu_0 - \beta^2 - k_z^2$, the attenuation along x decreases with increasing H. That is, increasing H the higher order modes will be less attenuated and more of them will still be detectable at a given distance x from the ferrite. At a given distance x we then expect the effect of higher order modes to be the more significant the more the applied magnetic field is increased. In fact we find that by increasing H the oscillations detected along y become always more complex (see Fig. 6).

3) The higher is the order of the mode and the higher can be shown to become the attenuation in the x direction. We expect, therefore, for a given H, the number of modes detectable to decrease with x, namely the pattern of the field along y to become the more simple and finally to approximate a pure exponential decay (when only the zero-order mode is significant) the more one moves away from the ferrite slab. This effect can be easily seen in Fig. 5.

4) Figure 7 suggests the presence of second- and fourth-order modes but, with the shape and length of the probe used in taking the measurements reported in Fig. 4 the contribution of the fourth-order mode would roughly cancel out and only the zero- and second-order modes would be detected. Let then the signal detected be of the form:

$$|E| = \left[C + D \sin \frac{2\pi}{b} z \cos(\beta_2 - \beta_0) y\right]^{1/2} \quad (1)$$

wher C and D are the amplitudes of the zero- and second-order modes. If $C/D \ll 1$ (1) reduces to:

$$|E| \simeq C^{1/2}\left[1 + \frac{D}{2C} \cdot \sin \frac{2\pi}{b} z \cos(\beta_2 - \beta_0) y\right]. \quad (2)$$

If we take for β the values given Barzilai and Gerosa [2] for the zero- and second-order modes, i.e., we take $\beta_0 = 1.9\, \omega\sqrt{\mu_0\epsilon_0}$ and $\beta_2 = 4.4\omega\sqrt{\mu_0\epsilon_0}$, we find for the wavelength of the cosinusoidal pattern $\Delta\lambda = 2\pi/(\beta_2 - \beta_0) = 1.2$ cm, in fairly good agreement with the value of wavelength we can measure on the pattern shown in Fig. 4.

ACKNOWLEDGMENT

It is a pleasure to thank Prof. G. Barzilai for many helpful discussions.

M. BUJATTI
Universita di Roma
Facdta di Ingegneria
Via Eudussiana
Rome, Italy

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